International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 6 Issue 8 Ver. II ǁ 2018 ǁ PP. 50-53

Research Status Of Curve Smoothing Algorithm

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ABSTRACT *: With the continuous development of computer technology, the application of curve and surface modeling technology in modern industrial product design and manufacturing process is more and more extensive. Because of the errors in design and measurement, the smoothness of curves can not meet the requirements of design and manufacture. Curve smoothness not only affects the appearance of the product, but also directly affects the degree of difficulty in manufacturing process and the mechanical properties of the product. It is an important sign to evaluate the quality of the product. The interpolation and approximation construction methods of curves, the definition of fairing, fairing criteria and fairing methods are discussed. The relationship between curve fairing and surface fairing is introduced.*

KEY WORDS *: Curve fairing,Fairing method,Fairing criterion,Geometric modeling*

Date of Submission: 15-11-2018 Date of acceptance: 29-11-2018

I. INTRODUCTION

Computer Aided Design, (CAD) and Computer Aided Manufacturing, (CAM)Originated in the aviation industry, due to the complex shape of aircraft and the large number of free curves and surfaces, CAD/CAM technology has been closely linked with the free curve and surface modeling technology from the beginning. Curve and surface modeling technology was established by Coons, Bezier and other masters in 1960s[1].After years of research and development, curve and surface modeling technology has formed a geometric theory system based on NURBS curve and surface parametric design and implicit algebraic curve and surface representation, with interpolation, approximation, fitting and other means as the skeleton[2].

With the development of industry, because NURBS method can accurately represent quadratic regular curves and surfaces, can be easily controlled and implemented by weight factors, and can be directly promoted in the thinking space, in 1991, the International Organization for Standardization (ISO) took NURBS as the only mathematical method to define the geometric shape of industrial products. Since then, the NURBS method has become the most important foundation in the development trend of curve and surface modeling technology[3].

II. INTERPOLATION AND APPROXIMATION OF CURVES

Interpolation first appeared in the field of numerical analysis, usually understood as generating new data from known data. In curve construction, a smooth curve is generated by giving some data points. Approximation also generates new data through known data. The difference between them is that the curves generated by interpolation method pass through all given points, which are called data points. The curves generated by approximation method do not necessarily pass through all points. These points are called control points[4].

Interpolation is one of the important methods of function approximation. It can estimate the approximation of function at other points by the value of function at finite points. That is to say, on the basis of discrete data, continuous function is interpolated so that the continuous curve can pass through all given discrete data points.

The interpolation problem can be described as:The value of a real-valued function on a given interval $[a,b]$ at $n+1$ points which is different from each other in the interval is required to estimate the value of the function at a point in the interval. The method is: in a pre-selected function class consisting of a simple function and a parameter, the function satisfying the condition is obtained and used as the evaluation. This is called interpolated function, called interpolation node, called interpolation function class. The above equation is called interpolation condition. The function satisfying the above equation is called interpolation function. The above solution problem is called interpolation problem.

There are many interpolation methods, the most common one is polynomial interpolation, including Lagrange interpolation polynomial and Newton interpolation polynomial, in addition to Hermite interpolation, piecewise interpolation, spline interpolation and trigonometric interpolation.

When there are too many data points, it is very difficult to construct interpolation function to make it pass through all data points. Sometimes, even if such function can be constructed, the efficiency of the whole algorithm will be reduced because of the high number of functions or the complex form. In practical application, it will be more difficult to find the expression of analytic function, which is objective. In other words, because there are too many data points, the impact of measurement errors on data is also increased. At this time, we do not need to find an interpolation function to accurately pass through all data points. Therefore, we often choose a lower number of functions, in a sense, the best approximation of these data points.

The essence of approximation problem is the approximate representation of functions, that is, finding a function $p(x)$ in a selected class of functions, making it an approximate representation of a known function $f(x)$ in a certain sense, and finding out the error caused by the approximation of $f(x)$ by $p(x)$. This is the approximation problem of functions. Among them, there are still various ways to determine the approximate expression of function $p(x)$ which is used to approximate the known function $f(x)$. The approximation degree of function $p(x)$ for $f(x)$, that is, the error, can also have various meanings. So the formulation of function approximation problem has various forms, and its content is very rich.

III. SMOOTHING OF CURVES

What is "fairing"? "Fairing", as the name suggests, is smooth and pleasing to the eye. "Smoothness" and what we call "smoothness" in our daily life are two different concepts, which can not be confused. "Smoothness" usually refers to the parametric continuity or geometric continuity of a curve. It is a mathematical term. And "smoothing" has not only the requirement of continuity in mathematics, but also the requirement of function (such as aesthetics, mechanics, NC machining). Because it involves the aesthetics of geometric shape and is influenced to a great extent by people's subjective factors, in general, "smoothness" is still a vague concept without an accurate definition and unified standard. Now, can we not smooth the curve? The answer is No. If there is no inherent rule for smoothing, how can we judge the smoothness of curves? Thus, the key to the problem is not whether fairness has objectivity, but how to coordinate the relationship between the objectivity and uncertainty of fairness, which is the problem to be solved by the definition and criterion of fairness.

Two problems need to be solved for smoothing curves[5] :

(1) what kind of curve is smooth, that is, fairing criteria;

(2) For unfaired curves, which mathematical treatment should be adopted to satisfy or improve their fairness, that is, fairing treatment method.

To smooth curves, we need to give specific fairing criteria first. Here are some fairing criteria which are often used in fairing processing.

(1)for plane curves

fairing criteria1(Farinproposed)[6] :

For a curve, if its corresponding curvature curve is continuous, has appropriate symbols (if the concavity and convexity of the curve are known), and is as close as possible to a piecewise monotone function with as few monotone segments as possible, then the curve is considered monotone.

fairing criteria2(Su Buqing,Liu Dingyuan proposed)[7] :

(a)The two order parameter is continuous (C2 continuous).

(b)There are no additional inflection points.

(c) The curvature changes are more uniform.

fairing criteria3(Shi Fazhong proposed) [8] :

(a)The two order geometric continuum (refers to the position, tangent direction and curvature vector continuous, as G2).

(b) There are no singular points and unnecessary inflection points.

(c)The curvature changes more evenly.

(d) Strain energy is small.

(2)For space curve

The following criteria are proposed in document [9]:

(i) Two order smoothness;

(a) The two order vector of a curve is continuous, and the curvature is continuous.

(b)The curve (quadratic) of the low order spline may have a jump in the curvature of the node, which requires the jump degree and the minimum possible.

$$\sum\left|k\left(t_i^+\right)-k\left(t_i^-\right)\right|<\varepsilon \qquad (1)$$

(ii)There is no excess inflection point.

(a)There should be G inflection points in the curve, while there are more than G inflection points in fitting (interpolation and approximation).

(b)There should be an inflection point where there should be no turning point.
(iii)The curvature changes more evenly.
(iv)There is no redundant variable deflection point (point whose deflection is zero, usually related to the point whose deflection is variable), that is to say, the following situation is not allowed:
(a)There should be H deflection points, and there are redundant H deflection points when fitting (interpolation and approximation).
(b)There should be no turning point where there should be no turning point.
(v) Uniform variation of torsion
(a) The torsion may be discontinuous at the node, and the leaping and small enough should be made at this time.

$$\sum\left|\tau\left(t_i^+\right)-\tau\left(t_i^-\right)\right|<\varepsilon \qquad (2)$$

(b)The variation of torsion is uniform, and there is no continuous sign change.

At present, according to the number of type points (or control points) modified each time, curve fairing algorithms can be divided into global fairing and local fairing. The global fairing algorithm is represented by least squares method and energy method. This kind of fairing algorithm can reduce the curvature of the curve as a whole and make the curvature of the curve change more uniformly, but the shape of the faired curve changes greatly compared with the original curve, and the direction of deformation can not be controlled. In the local fairing algorithm, the point selection modification method and curvature method are used. Rate method is representative. This kind of smoothing method has better smoothing effect in a small local range, but it can not eliminate the concave defects.

(1)Global fairing
(a) least square method
In curve reconstruction, least squares method is the most important method. Its basic principle is to consider the deviation of approximate function $\varphi(x)$ from $\delta_i=\varphi(x_i)-y_i \quad (i=0,1,\text{L} \ ,m)$ on given value point $(x_i, y_i)(i=0,1,\text{L} \ ,m)$ as a whole, and to make it the smallest according to a certain measure standard.

1.Minimize $\max\limits_{0\le i\le m}\left|\varphi(x_i)-y_i\right|$

2.Minimize $\sum\limits_{i=0}^{m}\left|\varphi(x_i)-y_i\right|$

3. Minimize $\sum\limits_{i=0}^{m}\left|\varphi(x_i)-y_i\right|^2$

(b)energy method

In 1969, Hosaka[10] proposed a curve smoothing method based on the energy extremum principle, called energy method. The basic idea of this method is to use the cumulative chord length cubic spline as the reconstruction curve and the total energy of the spline as the objective function. Its fairing process is to solve the extremum problem of the objective function. The mechanical model of this method is very intuitive. Assuming that the sequence of shape points before and after smoothing a curve is $M_i\ (i=0,1,\text{L} \ ,n)$ and $N_i\ (i=0,1,\text{L} \ ,n)$, the formula for calculating strain energy is as follows:

$$E=\frac{1}{2}\alpha\int k^2dx+\frac{1}{2}\sum_{i=0}^{n}\beta_i\left\|N_i-M_i\right\|^2 \qquad (3)$$

In (3), α ——Stiffness coefficient of the curve;
β_i——Elastic coefficient;
k——Curve curvature.

(2)Local fairing

Two commonly used global fairing algorithms are introduced. Both least squares method and energy method are suitable for the case of relatively large number of non-fairing points. However, when there are few non-fairing points (such as only a little non-fairing), the above two methods will undoubtedly result in a large amount of computation and speed of operation. Slow down. Several common local smoothing algorithms are introduced below.

(a)Point revision method
The smoothing process of the point selection method is: Step1: find out the "bad points" one by one; Step2: modify the "bad points". These two processes are discussed below.
Step1:Discrimination of "bad points"
There are two methods to distinguish "bad points": (i) Users decide "bad points" by themselves according to their observations, which is called interactive method. (ii) According to the corresponding fairing criteria, a criterion

for judging "bad points" is established to accurately identify all "bad points", which is called automatic method. $\boldsymbol{Q}_i\left(i=0,1,\mathrm{L}\ ,n\right)$ is assigned to the set value, assuming that $\boldsymbol{Q}\left(t\right)$ is the curve of interpolation and $\left\{\boldsymbol{Q}_i\right\}$, and k_i is the relative curvature at the point of type value. For automatic methods, in general, we use the following "bad points" criteria:

1)The k_i discontinuous type value points are called the 1 kind of bad points.

2)In the sign sequence $\left\{sign\left(k_i\right)\right\}$ of curvature, the point $\boldsymbol{Q}_i$ of continuous sign change is called two kinds of bad points even if the condition $k_{i-1}k_i<0$ and $k_ik_{i+1}<0$ hold.

3)The points of continuous sign change in the first order difference symbol sequence $\left\{sign\left(\Delta k_i\right)\right\}$ of k_i, even if condition $\Delta k_{i-1}\Delta k_i<0$ and $\Delta k_i\Delta k_{i+1}<0$ hold, are called three kinds of bad points.

Step2:Modification of "bad points"

After identifying the "bad points", we need to modify them. At present, the commonly used "bad point" modification methods include kejellander method [11] [12], node deletion and insertion method [13] [14], roundness method and base spline method. The advantage of this method is that the criterion is simple, local modification, fast calculation speed and fairing effect is very good in the case of fewer "bad points". The disadvantage of this method is that when there are many "bad points" in succession, the criterion of "bad points" depends on the curvature of adjacent points to calculate, so the criterion will appear certain. The degree of misjudgement is not very good.

(b)Curvature method

The objective function of energy smoothing is to calculate the square and integral of curvature. The objective of smoothing is to reduce the strain energy of the curve as much as possible, that is, to reduce the curvature of the curve, so that the curve area is smooth and the shape change is large.

Chen Liang [15] developed a curve smoothing method based on the curvature method of wavelet decomposition, extracted the part with lower frequency after wavelet decomposition as a new curvature map, and finally reconstructed the smoothed product contour curve. The basic idea of these methods is to calculate the curvature map of the curve, then smooth the curvature map of the curve, not directly smooth the curve. Then, according to the curvature map after smoothing and combining with the original value points, the original curve can be inversely calculated, so that the fairing curve of the original curve can be obtained.

IV. CONCLUSION

The development of curve smoothing technology provides powerful support for modern manufacturing industry. Curve modelling technology extends from traditional curve interpolation, approximation and fitting to various smoothing technologies, resulting in the phenomenon of multi-disciplinary and multi-field cross-integration. The CAD industry is also rapidly putting new technologies and theories into industry applications, and the importance of visible smoothing technology to modern industry. Throughout the development of curve modelling technology, finding a better curve fairing technology is the key to promote the whole CAGD research.

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Jiang Mingyang "Research Status Of Curve Smoothing Algorithm " International Journal of Research in Engineering and Science (IJRES), vol. 06, no. 08, 2018, pp. 50-53